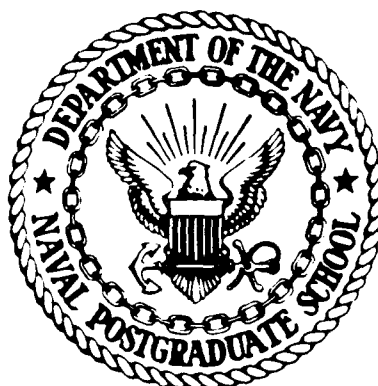


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THESIS

AN ANALYTICAL
HIGH VALUE TARGET ACQUISITION MODEL

by

Kevin J. Becker

March 1986

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An Analytical High Value Target Acquisition Model

by

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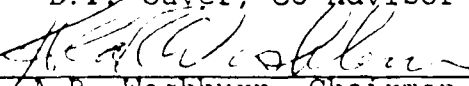
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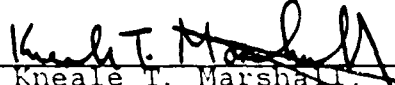

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ABSTRACT

An analytical High Value Target (HVT) acquisition model is developed for a generic anti-ship cruise missile system. The target set is represented as a single HVT within a field of escorts. The HVT's location is described by a bivariate normal probability distribution. The escorts are represented by a spatially homogeneous Poisson random field surrounding the HVT. Model output consists of the probability that at least one missile of a salvo acquires the HVT, conditioned on the number of missiles in the salvo which penetrate the HVT area defense. In addition, the fall of multiple penetrators is modeled using a conditional multinomial probability distribution. The model's equations are used to solve for an optimal missile seeker range gate, given a probability distribution describing the location of the HVT within the targeted formation at the time the missile commences its search. Included in an appendix is a time-dependent model describing HVT location which provides for HVT movement during missile time of flight up to the moment of active search.

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I. INTRODUCTION

With the advent of modern, sophisticated anti-ship cruise missiles (ASCM), opposing naval battle forces have an over-the-horizon (OTH) strike potential which, when actualized by appropriate tactics, will herald a new era in naval combat. As an opening remark the above may sound dated, for the cruise missile has arrived and the technology is aboard many combat fleets of the world. Nevertheless, the implication of next-generation microelectronics and artificial intelligence aboard the ASCM of tomorrow presents the forward-looking military operations research analyst with promising opportunities for tactical improvement. Revolutionary advances in ASCM technologies will provide future systems with multi-target discrimination ability, reduced soft-kill susceptibility, and other significant improvements. Hence, tactical options should be evaluated well in advance of a new missile's arrival in the Fleet.

In an attempt to destroy an opposing, escorted high value target (HVT), the decision-maker possessing an ASCM OTH capability must choose when to strike, using appropriate force (e.g., total salvo size), a decision surely influenced by targeting accuracy, intelligence regarding opposing force composition, and first-strike criticality. Quantitative answers to the above decision problem are by nature probabilistic ones. The decision-maker can be assisted by an associated estimate of HVT kill probability from a model.

The multi-target scenario has been modeled quite extensively using elaborate Monte-Carlo computer simulation techniques which, even after considerable programming input effort and program execution time, yield estimates that often neglect the essential vagaries of the war-at-sea environment. Simulation has contributed much insight to the

decision problem at hand, but is generally inappropriate as a tactical decision-aid because of its inflexibility. The so-called Bernoulli Trials model is a much simpler, purely analytical probability model which has been employed as a tactical decision-aid, [Ref.1].

In the Bernoulli Trials model, HVT defense is essentially limited by the number of available fire-control channels, each of which is awarded a deterministic or probabilistic number of ASCM kills. Targeting information is assumed to be accurate and all ASCM that penetrate the HVT area defense have equal probability of acquiring any of the multiple targets present. In an extension to the model, HVT acquisition by the ASCMs is biased upward, thereby endowing the missiles with multi-target discrimination ability. The model's equations are solved iteratively for an estimate of salvo size needed to inflict a firepower-kill on the escorted HVT with a specified probability of success. The driving parameters are the number of escorts, the effectiveness of HVT defense, and the required number of hits on the HVT to achieve a firepower-kill.

The Bernoulli Trials model and its simulation counterparts make simplistic assumptions regarding ASCM target acquisition. An accurate target acquisition model is an essential part of any model that attempts to provide the strike planner with a cost-effective estimate of requisite salvo size, where "cost" refers to the number of ASCMs employed. (While it is true that the prudent tactician may wish to launch "extra" missiles as a hedge, it is also likely that missiles will be in short supply; hence, he needs to know what basic number he is supplementing with "extras".) This paper offers a new, purely analytical HVT acquisition probability model designed to enhance existing multi-target ASCM OTH models. It retains much of the simplicity of the Bernoulli Trials model, but:

- Reflects statistical fluctuations in targeting accuracy in a better way.

- Incorporates the attacker's missile search area and guides his range gating decisions.

II. THE MODEL

A. DESCRIPTION

1. Overview and General Assumptions

The ANALYTICAL HVT ACQUISITION MODEL estimates the probability that at least one penetrator (an ASCM that survives the HVT area defense) acquires a single HVT within an escort field of low value targets (LVT). Additionally, the expected number of penetrators that acquire the HVT and her escorts (ie., the fall of shot) is estimated. The model's outputs serve as reasonable Measures of Effectiveness of the decision-maker's choices under uncertainty; namely, given the current tactical picture and opportunity to strike at the enemy, how many missiles should he employ and how are they best programmed to achieve a desired probability of HVT acquisition?

The choices of launch bearing and ASCM search programming are influenced by the accuracy and time late of the targeting information and the missile time of flight to the target area. Where and when opportunities exist, analysis of the offense's information state concerning the targeted formation may be profitable even when information is scarce and uncertain. The model assumes the following information is available to the strike planner:

- A bivariate normal probability density function (or other appropriate probability density) describing the location of the HVT within an array of escorts.
- An estimate of LVT density about the HVT.
- An appropriate set of missile seeker sensor Sweep Widths.
- An estimate of target acquisition probability, given target detection.

- A simplified estimate of ASCM reliability during the search, detection, and acquisition phases of the attack.
- A velocity-time random vector of HVT/escort field motion (optional).

The missile firing strategy used in the model is that of near-simultaneous firings (a short firing interval with respect to appreciable target movement) on a common launch bearing calculated to intercept the mean of the HVT location density. If the model user has a postulated motion vector of the enemy formation, the HVT density can be updated to incorporate the time delay from the generation of the previous targeting ellipse to the time of arrival of the salvo at the search area, in which case the salvo launch bearing should be calculated to intercept the mean of the updated HVT density at the time of missile search. Adjusting the HVT density and launch bearing becomes necessary when missile time of flight and enemy motion result in significant displacement of the enemy formation from its location at the time of launch.

2. Modeling the ASCM System

A generic ASCM system is modeled. Missile flight is straight down the launch bearing through the programmed search area. The assumption of zero missile navigation error is made since incorporation of the relatively small errors associated with a highly accurate system is not apt to heavily influence model results. (Deference is made however, to the exacting navigational accuracy required in especially long range firings.) The missile seeker sensing device is essentially off (ie., receives no information), prior to ASCM arrival at the search area. An important assumption concerning the offensive firing is the notion of conditional independence between shots; each missile within a salvo is given the same launch bearing and search program, but retains functional independence (autonomy) with regard to target acquisition.

The effective ocean area searched out by the ASCM is a function of operator input (range gate) and the missile seeker sensor Sweep Width(s) given by a Definite Range Law of Detection. The resulting area is a rectangle (or set of rectangles when multiple Sweep Widths are defined). In the existing detection environment, individual targets may be grouped by various homogeneous characteristics (e.g., size, active or passive signatures, shape, motion, etc.). For the purpose of detection, similar targets are subject to a common sensor Sweep Width. The model may be used in the situation where the HVT possesses a detectable feature that distinguishes it from the LVT class. For example, a sensor's Sweep Width for an aircraft-carrier may be wider than its Sweep Width for a destroyer-escort because of the carrier's relatively large size.

A target is detected if it lies within its associated rectangle of effective search and is "swept" by the missile seeker sensor (ie., is in the sensor's field of view and within half a Sweep Width of the search axis). The model assumes the flightpath and the search axis of each missile are the same. The model's generic ASCM accomplishes its search in a matter of seconds, making search analogous to taking a snapshot photograph (an aggregated detection process). Consequently, target motion is treated as negligible throughout successive searches by individual ASCM; a reasonable assumption if the interarrival times between missiles within the salvo are small. The assumption is made that each of the reliable ASCM in the salvo detect the same set of targets. This is because of their identical search programming, the Definite Range Law of Detection, and a short firing interval.

Once a target or number of targets have been detected, the missile enters an acquisition phase in which a decision to acquire a particular target is made. The model

associates a conditional probability of acquisition, an input, with each target (HVT and one or more LVTs) detected by the ASCM. Implicit in the detection and acquisition modeling is the assumption that, for purposes of acquisition, detected targets are processed independently and in sequence from near to far within the missile seeker sensor's range gate. Therefore, acquisition of a particular target is influenced by two factors following detection: (a) its relative position within the range gate, and (b) its associative conditional probability of acquisition. An ASCM detection/acquisition system with a multi-target discrimination ability has a higher probability of acquiring the HVT than that of acquiring a LVT nearby. The model represents this discrimination by incorporating multiple Sweep Widths and conditional acquisition probabilities associated with the ASCM system and target set. In practice, Sweep Widths and conditional acquisition probabilities may be difficult to estimate. Possible sources are ASCM system simulations, observational test firings, and expert opinion.

One final assumption is made concerning the nature of the offensive firing. Destruction of defensive firepower is the initial result of missile hits on the HVT and her escorts. Target sinkings occur long after the ASCM salvo has run its course. Hence, although hits on secondary targets degrade defensive firepower, total target elimination is not likely to occur during the search and detection/acquisition phase of the ASCM attack (ie., an HVT or LVT that is present for detection by the first missile in the salvo will be present for the last missile to detect).

3. Modeling the Target Formation

The target formation is modeled as a single HVT within a bounded, spatially homogeneous Poisson random field of escorts. The Poisson field completely characterizes the number of LVT in the field (a Poisson random variable given

by the field density parameter) and the placement of LVT within the field (uniformly randomly distributed conditioned on the number). It may seem that the Poisson field is better suited for modeling a scattering of merchant shipping in a sea lane, rather than a geometrically rigid Anti-air Warfare (AAW) screen. In reality, screens are not rigid: escort ships are randomly patrolling sectors, exchanging stations, detaching to form Anti-Submarine Warfare (ASW) groups, etc., making for a dynamic structure in contrast with the mental image of a rigidly symmetric AAW screen. A desirable feature of the Poisson field is the notion that the number of LVT in the field is a random variable. This is likely to be true if the strike planner's intelligence estimate of enemy force composition is uncertain and targeting of secondary LVT is unavailable. The average number of escorts accompanying the HVT and the area of the Poisson field can be estimated with the aid of intelligence reports, electronic support measures, and subjective probability encoding, thereby furnishing a reasonably good estimate of the field density parameter. For example, a field of 8 escorts uniformly distributed within a circle of radius 12 nautical miles about the HVT has a density of 0.0177 LVT per square nautical mile. As will be seen, such a figure can be utilized to select a nearly optimum range gate setting.

B. ANALYTICAL DEVELOPMENT

1. Geometry

A few assumptions concerning the orientation of the Poisson field of escorts and the effective search area of the missiles are required in order to obtain analytical results:

- The HVT is centrally located within the escort field.
- The escort field area is large compared to the ASCM search area (ie., the search area is wholly contained in the field).

The model's analytics reference a standard xy-coordinate system oriented so that the y-axis lies on the missile flight path. The origin represents both the estimated mean of the HVT density in residence above the xy-plane and the center of the Poisson field of escorts. The effective search area of the missiles is dimensioned by range gate (measured along the y-axis) and Sweep Width (measured along the x-axis). It is important to remember that the effective search area can be a group of rectangles with different Sweep Widths for different target classes when a distinction is made, in which case each rectangle of effective search has a common y-dimension (range gate) and a unique x-dimension (Sweep Width). Figure 2.1 is a sketch of model geometry.

2. Single Shot Penetrator

For illustrative simplicity, the HVT true location density at the time of missile search is assumed to be circular normal. Appendix A contains a time-dependent general bivariate normal HVT location density as an alternative.

Let the random variable Y represent the y-coordinate of the HVT's true location, measured from its estimated location (ie., the origin). In the language of Bayesian statistics, the distribution of Y is the posterior distribution of the HVT's true y-coordinate, given a point estimate of its y-coordinate of $y = 0$. In the following discussion, y refers to a possible value for the random variable Y . (The same interpretation holds for the random variable X , the HVT's true x-coordinate.)

The single shot calculation requires the following input constants and probabilities:

- λ = LVT escort field density.
- W_l = Sweep Width governing LVT.
- W_h = Sweep Width governing HVT.

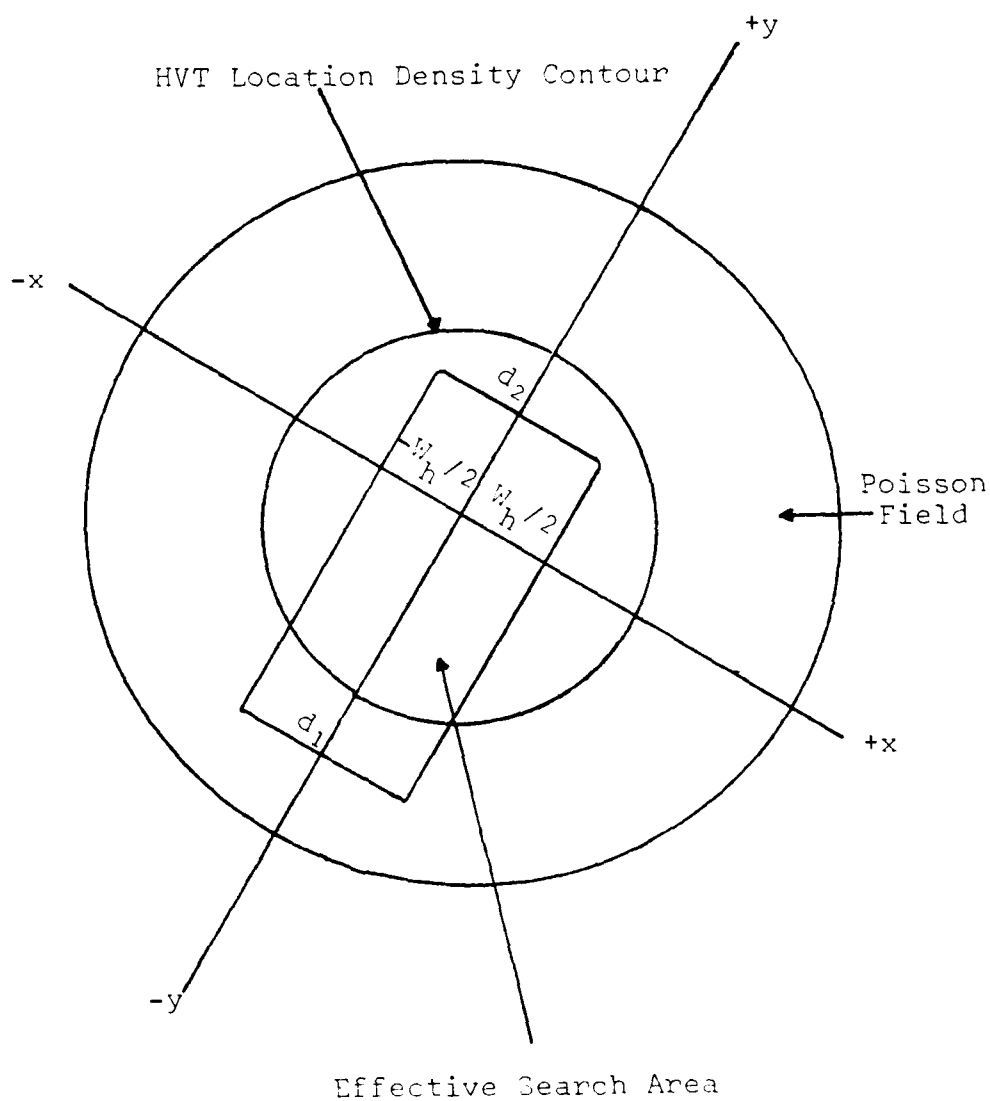


Figure 2.1 MODEL GEOMETRY.

- d_1 = Near edge of range gate (y coordinate).
- d_2 = Far edge of range gate (y coordinate).
- r = Missile reliability.
- p_ℓ = Conditional probability of acquiring a LVT given LVT is detected.
- p_h = Conditional probability of acquiring the HVT given HVT is detected.

Assume that the HVT is contained in the ASCM's effective search area. Since detected targets are processed from near to far within the missile seeker sensor's range gate, HVT acquisition requires that no LVT has been acquired (ie., selected and homed-on) in the region $W_\ell(y-d_1)$. By virtue of the Poisson field assumption this is equivalent to experiencing a free path of area $W_\ell(y-d_1)$ in a two-dimensional filtered Poisson process of rate λP_ℓ .

Define the following events:

D: HVT acquisition.

E: Filtered escort free path to the HVT.

The probability of HVT acquisition is expressed as

$$P(D) = \int_{d_1}^{d_2} \int_{-W_h/2}^{W_h/2} r p_h P(E|Y=y) f(x,y) dx dy, \quad (2.1)$$

where

$$f(x,y) = e^{-(x^2+y^2)/2\sigma^2} \frac{1}{2\pi\sigma^2}, \quad (2.2)$$

is the probability density function describing the location of the HVT, and

$$P(E|Y=y) = e^{-\lambda W_\ell P_\ell (y-d_1)}, \quad (2.3)$$

is the conditional probability of event E, given the true range coordinate of the HVT. In terms of these components,

$$\begin{aligned}
P(D) &= \int_{d_1}^{d_2} \int_{-W_h/2}^{W_h/2} r P_h e^{-\lambda W_\ell P_\ell (y-d_1)} e^{-(x^2+y^2)/2\sigma^2} dx dy, \\
&= r P_h e^{\lambda W_\ell P_\ell d_1} \{ \Phi(W_h/2\sigma) - \Phi(-W_h/2\sigma) \} \\
&\quad \times \int_{d_1}^{d_2} e^{-\lambda W_\ell P_\ell y} e^{-y^2/2\sigma^2} \frac{dy}{\sqrt{2\pi}\sigma}.
\end{aligned}$$

The integral is evaluated by completing the square in the exponent and adjusting the limits of integration.

Let $\kappa = \lambda W_\ell P_\ell$, $z = \frac{y+\kappa\sigma^2}{\sigma}$, and $dz = \frac{dy}{\sigma}$;

$$\begin{aligned}
P(D) &= r P_h e^{\kappa d_1 + \frac{1}{2}(\kappa\sigma)^2} \Delta\phi_x \left\{ \Phi\left(\frac{d_2+\kappa\sigma^2}{\sigma}\right) - \Phi\left(\frac{d_1+\kappa\sigma^2}{\sigma}\right) \right\}, \\
&= r P_h e^{\kappa d_1 + \frac{1}{2}(\kappa\sigma)^2} \Delta\phi_x \Delta\phi_y', \quad (2.4)
\end{aligned}$$

where

$$\Delta\phi = \int_a^b \phi(u) du$$

is the integral of the standard normal probability density function evaluated at the upper and lower limits shown.

3. Multiple Penetrators

The probability that at least one of m penetrating missiles acquires the HVT is computed by conditioning on the number of LVT in $W_\ell(y-d_1)$ of the effective search area. The solution is reached by integrating the HVT density over the effective search area, after first removing the condition on the number of LVT "in the way".

Let N , a random variable, be the number of missiles that acquire the HVT. The conditional probability that at least one of m missiles acquires the HVT is:

$$P(N>0|L=i, Y=y) = 1 - (1 - rP_h(1-P_\ell)^i)^m \quad (2.5)$$

where L is the number of LVT in $W_\ell(y-d_1)$.

The condition on LVT is removed using the knowledge that the assumed distribution of LVT escorts is Poisson with spatial density parameter λ . Expansion of the binomial term and rearrangement shows that

$$\begin{aligned} P(N>0|Y=y) &= 1 - \sum_{i=0}^{\infty} \left[(1 - rP_h(1-P_\ell)^i)^m \right. \\ &\quad \times e^{-\lambda W_\ell(y-d_1)} \frac{(\lambda W_\ell(y-d_1))^i}{i!} \Bigg], \\ &= 1 - \sum_{i=0}^{\infty} \sum_{k=0}^m \left[\binom{m}{k} (-1)^k (rP_h(1-P_\ell)^i)^k \right. \\ &\quad \times e^{-\lambda W_\ell(y-d_1)} \frac{(\lambda W_\ell(y-d_1))^i}{i!} \Bigg], \end{aligned}$$

$$\begin{aligned}
&= 1 - \sum_{k=0}^m \left[\binom{m}{k} (-rP_h)^k e^{-\lambda W_\ell (y-d_1)} \right. \\
&\quad \times \left. \sum_{i=0}^{\infty} \frac{((1-P_\ell)^k)^i (\lambda W_\ell (y-d_1))^i}{i!} \right], \\
&= 1 - \sum_{k=0}^m \binom{m}{k} (-rP_h)^k e^{-\lambda W_\ell (y-d_1) (1 - (1-P_\ell)^k)}.
\end{aligned}$$

(2.6)

Finally, to remove the condition on the location of the HVT, equation (2.6) is multiplied by the HVT location density and integrated using the same technique employed for the single shot calculation.

$$\text{Let } \delta(k) = \lambda W_\ell (1 - (1-P_\ell)^k);$$

$$\begin{aligned}
P(N>0) &= 1 - \int_{d_1 - W_h/2}^{d_2 - W_h/2} \sum_{k=0}^m \left[\binom{m}{k} (-rP_h)^k e^{-\delta(k)(y-d_1)} \right. \\
&\quad \times \left. e^{-\frac{(x^2+y^2)}{2\sigma^2}} \frac{dx dy}{2\pi\sigma^2} \right], \\
&= 1 - \Delta\phi_x \sum_{k=0}^m \left[\binom{m}{k} (-rP_h)^k e^{\delta(k)d_1} \right. \\
&\quad \times \left. \int_{d_1}^{d_2} e^{-\delta(k)y} e^{-y^2/2\sigma^2} \frac{dy}{\sqrt{2\pi}\sigma} \right],
\end{aligned}$$

$$= 1 - \Delta\phi_x \sum_{k=0}^m \binom{m}{k} (-rP_h)^k e^{\delta(k)d_1 + \frac{1}{2}(\delta(k)\sigma)^2} \Delta\phi_y',$$

$$= - \Delta\phi_x \sum_{k=1}^m \binom{m}{k} (-rP_h)^k e^{\delta(k)d_1 + \frac{1}{2}(\delta(k)\sigma)^2} \Delta\phi_y',$$

(2.7)

where

$$\Delta\phi_y' = \phi\left(\frac{d_2 + k\sigma^2}{\sigma}\right) - \phi\left(\frac{d_1 + \delta(k)\sigma^2}{\sigma}\right).$$

Expressions (2.4) and (2.7) allow the analyst to determine an optimal near edge of range gate setting, d_1 , ie. one that maximizes HVT acquisition probability, given the various operational parameters of the ASCM.

4. The Fall of Multiple Penetrators

More information can be obtained from the conditional distribution of the fall (distribution) of multiple penetrators, given that the HVT is contained in the missile's effective search area. Fall of shot terminology (e.g., dispersion) is normally associated with the firing assessment of ballistic projectiles, but can be adopted for ASCM if one can estimate the probability of "absorption" of an ASCM into each target class. Simply stated, a missile which penetrates the HVT area-defense will either have the HVT in its projected search area or not. If not, it is no longer a threat to the HVT, though there is a probability, usually small, that it will home on an escort. For the missiles that are a threat to the HVT, it is possible to estimate how they are distributed between HVT, escorts, and misses.

The acquisition decision of a single, conditionally independent penetrator is likened to a single trial of a multinomial experiment which, given the HVT is in the effective search area, has as its possible outcomes:

- A: ASCM acquires a LVT on the near-side of the HVT.
- B: ASCM acquires the HVT.
- C: ASCM acquires a LVT on the far-side of the HVT.
- D: ASCM fails to acquire a target.

The probability formulas of the above events are initially stated with the condition of the HVT's y-coordinate in the effective search area.

Let $\Delta d = d_2 - d_1$, and condition on $Y=y$; then

$$P(A|Y=y) = r(1 - e^{-\kappa(y-d_1)}), \quad (2.8)$$

$$P(B|Y=y) = rP_h e^{-\kappa(y-d_1)}, \quad (2.9)$$

$$P(C|Y=y) = r(1-P_h)(e^{-\kappa(y-d_1)})(1 - e^{-\kappa(d_2-y)}), \quad (2.10)$$

$$P(D|Y=y) = (1-r) + r(1-P_h) e^{-\kappa \Delta d}. \quad (2.11)$$

Derivations of the mean, variance, and covariance of the multinomial distribution's binomially distributed marginal distributions are given in many basic probability texts. Removing the condition of the y-coordinate of the HVT within the effective search area and normalizing with respect to y produces the following marginal expectations of the conditional multinomial events (ie., the expected number of missiles absorbed by each target class, given the HVT is in the effective search area).

Let M_i be the random number of penetrators that are "absorbed" into cell i , where $M_A + M_B + M_C + M_D = m$. The expected value of M_i is mp_i , where

$$\begin{aligned}
 P_A = P(A) &= r - (1/\Delta\phi_y) \int_{d_1}^{d_2} r e^{-\kappa(y-d_1)} e^{-y^2/2\sigma^2} \frac{dy}{\sqrt{2\pi}\sigma}, \\
 &= r(1 - e^{\kappa d_1 + \frac{1}{2}(\kappa\sigma)^2} \Delta\phi_y' / \Delta\phi_y); \quad (2.12)
 \end{aligned}$$

$$\begin{aligned}
 P_B = P(B) &= (1/\Delta\phi_y) \int_{d_1}^{d_2} r P_h e^{-\kappa(y-d_1)} e^{-y^2/2\sigma^2} \frac{dy}{\sqrt{2\pi}\sigma}, \\
 &= r P_h e^{\kappa d_1 + \frac{1}{2}(\kappa\sigma)^2} \Delta\phi_y' / \Delta\phi_y; \quad (2.13)
 \end{aligned}$$

$$\begin{aligned}
 P_C = P(C) &= (1/\Delta\phi_y) \int_{d_1}^{d_2} \left[r(1-P_h) (e^{-\kappa(y-d_1)} - e^{-\kappa\Delta d}) \right. \\
 &\quad \left. \times e^{-y^2/2\sigma^2} \frac{dy}{\sqrt{2\pi}\sigma} \right], \\
 &= r(1-P_h) (e^{\kappa d_1 + \frac{1}{2}(\kappa\sigma)^2} \Delta\phi_y' / \Delta\phi_y - e^{-\kappa\Delta d}); \quad (2.14)
 \end{aligned}$$

$$\begin{aligned}
 P_D = P(D) &= (1-r) + (1/\Delta\phi_y) \int_{d_1}^{d_2} \left[r(1-P_h) e^{-\kappa\Delta d} \right. \\
 &\quad \left. \times e^{-y^2/2\sigma^2} \frac{dy}{\sqrt{2\pi}\sigma} \right], \\
 &= (1-r) + r(1-P_h) e^{-\kappa\Delta d}. \quad (2.15)
 \end{aligned}$$

The general procedure for computing a marginal variance is shown below, followed by the equation of the variance of event B; the number of HVT acquisitions, given the HVT lies within the effective search area. In general,

$$\text{Var}(Y) = E\{\text{Var}(Y|X)\} + \text{Var}\{E(Y|X)\}.$$

Therefore,

$$\begin{aligned}\text{Var}(M_i) &= E\{mp_i(1 - p_i(y))\} + \text{Var}\{mp_i(y)\}, \\ &= E\{mp_i(1 - p_i(y))\} + m^2 \left[E\{p_i^2(y)\} - (E\{mp_i(y)\})^2 \right], \\ &= m(m - 1)E\{p_i^2(y)\} + mE\{p_i(y)\} - m^2(E\{p_i(y)\})^2, \\ &= m(m - 1)E\{p_i^2(y)\} + mp_i - (mp_i)^2. \quad (2.16)\end{aligned}$$

Equations (2.8) through (2.9) can be substituted into equation (2.16) in place of $p_i(y)$ and the corresponding expressions (2.12) through (2.15) substituted in place of p_i . The expectation of $p_i^2(y)$ remains to be calculated and can be accomplished using

$$E\{g(p_i(y))\} = \frac{\int_{d_1}^{d_2} g(p_i(y)) \phi(y) dy}{\int_{d_1}^{d_2} \phi(y) dy},$$

the expectation of a function of $p_i(y)$, given the HVT is in the missile's effective search area (hence the normalization quotient).

Calculation of the variance of M_p , the number of ABOM that acquire the HVT, given the HVT is contained in the missile's effective search area is as follows;

$$\begin{aligned}
\text{Var}(M_E) &= m(m-1)(1/\Delta t_y) \int_{d_1}^{d_2} \left[r^2 P_h^2 e^{-2\kappa(y-d_1)} \right. \\
&\quad \left. \times e^{-y^2/2\sigma^2} \frac{dy}{\sqrt{2\pi}\sigma} \right] + mP_E - (mP_E)^2, \\
&= m(m-1)r^2 P_h^2 e^{2(\kappa d_1 + (\kappa\sigma)^2)} \Delta t_y'' / \Delta t_y \\
&\quad + mP_E - (mP_E)^2, \tag{2.17}
\end{aligned}$$

where

$$\Delta t_y'' = \frac{5(d_2 + 2\kappa\sigma^2)}{\sigma} - \frac{4(d_1 + 2\kappa\sigma^2)}{\sigma}.$$

It is important that the effect of the range gate decision variable on HVT acquisition probability can be explored using the model's analytical equations. An illustrative numerical example is presented in the next chapter.

III. OPTIMAL RANGE GATING

The near edge of the missile seeker sensor's range gate, d_1 , is an influential decision variable. As the range gate increases in length, more targets are likely to be detected and processed for acquisition, making the ASCM's task of selecting the HVT more difficult. As the range gate narrows, the probability of capturing the HVT in the effective search area (ie., detecting the HVT) decreases. If the far edge of the range gate, d_2 , has been chosen such that the subsequent choice of d_1 brackets the mean of the HVT targeting density along the missile flight path, equation (2.7) can be used to solve for a value of d_1 that maximizes the probability that at least one penetrating missile acquires the HVT, given m penetrators are realized. If an estimated distribution of the number of penetrators were available, the law of total probability could be used to obtain an unconditionally optimal or "weighted" value of d_1 based on the conditionally optimal values of d_1 obtained from graphing equation (2.7) for m penetrators, $m = 1, 2, \dots, s$, where s is the total salvo size. Or, in the situation where only an expected number of penetrators is available, equation (2.7) can be graphed once for the optimal value of d_1 based on the expected value of m . The numerical example that follows illustrates how the optimal choice of d_1 is affected by ASCM multi-target discrimination ability, the number of penetrators, and HVT targeting accuracy.

The example uses the following hypothetical input parameter values:

$$\begin{aligned}\sigma &= 3.0 \text{ Nm} \\ \rho &= 0.0177 \text{ LVT/Nm}^2 \\ W_{HVT} &= 6 \text{ Nm}\end{aligned}$$

$$\begin{aligned}
 W_{\ell} &= 6 \text{ Nm} \\
 P_h &= 0.97 \\
 P_{\ell} &= 0.4 \\
 r &= 0.99 \\
 d_2 &= +5.0 \text{ Nm}
 \end{aligned}$$

Figure 3.1 is a plot of a single shot penetrator's HVT acquisition probability versus d_1 , where zero on the horizontal axis is the expected location of the HVT along the flight path. The range gate $[d_1, d_2] = [-5.1, +5.0]$ maximizes the single shot probability of HVT acquisition. Figure 3.2 contrasts the curve of Figure 3.1 (curve A) with a curve generated from identical input parameter values excepting conditional acquisition probabilities, which have been set equal to 0.97 (curve B). The relatively gradual ramp of curve A is representative of a "more forgiving" missile (ie., one that has a better multi-target discrimination ability).

Figure 3.3 demonstrates the increase in HVT acquisition probability from firing a salvo and realizing four penetrators versus firing a single shot with the hope of realizing one. While it is difficult to spot a noticeable difference in d_1 associated with the maxima of each curve, the actual difference is about 1.7 nautical mile, with the four penetrators having the "longer" optimal range gate. The result is a consequence of maximizing the probability that at least one missile acquires the HVT. The marginal "single shot" probability of HVT acquisition of each missile is effectively reduced while the chance at least one acquires the HVT is increased. This phenomenon becomes more apparent in Figure 3.4, ten penetrators versus a single shot penetrator.

Figure 3.5 is a plot of number of penetrators versus HVT acquisition probability where the conditionally optimal range gate has been used at each increment of the indepen-

dent variable. The marginal return of the first few penetrators is striking. Figures 3.7 and 3.8 demonstrate the marked effect that an increasing HVT location error sigma has on HVT acquisition probability in both the single shot penetrator and salvo/multiple penetrator cases. As sigma increases, it becomes apparent that the simple firing strategy may very well be out-performed by a strategy that involves firing a spread of missile salvos (assets permitting), an interesting hypothesis for future research.

Table I summarizes conditional fall of shot expectation and standard deviation for a single shot penetrator and various numbers of multiple penetrators over selected values of sigma. Again, the fall of shot for events A, B, C, and D is conditioned on the HVT's containment and location within the missile's effective search area. As sigma increases, the optimal range gate opens, the probability of containing the HVT in the effective search area decreases, and, in the case of salvo/multiple penetrators, the conditional fall of shot expectation becomes more evenly distributed between the HVT and the LVT on the near-side. It should be noted that the total expected threat to the escorts cannot be determined by simply adding the expectation of events A and C, because there is a chance that some escorts may lie in the missile's effective search area though the HVT does not.

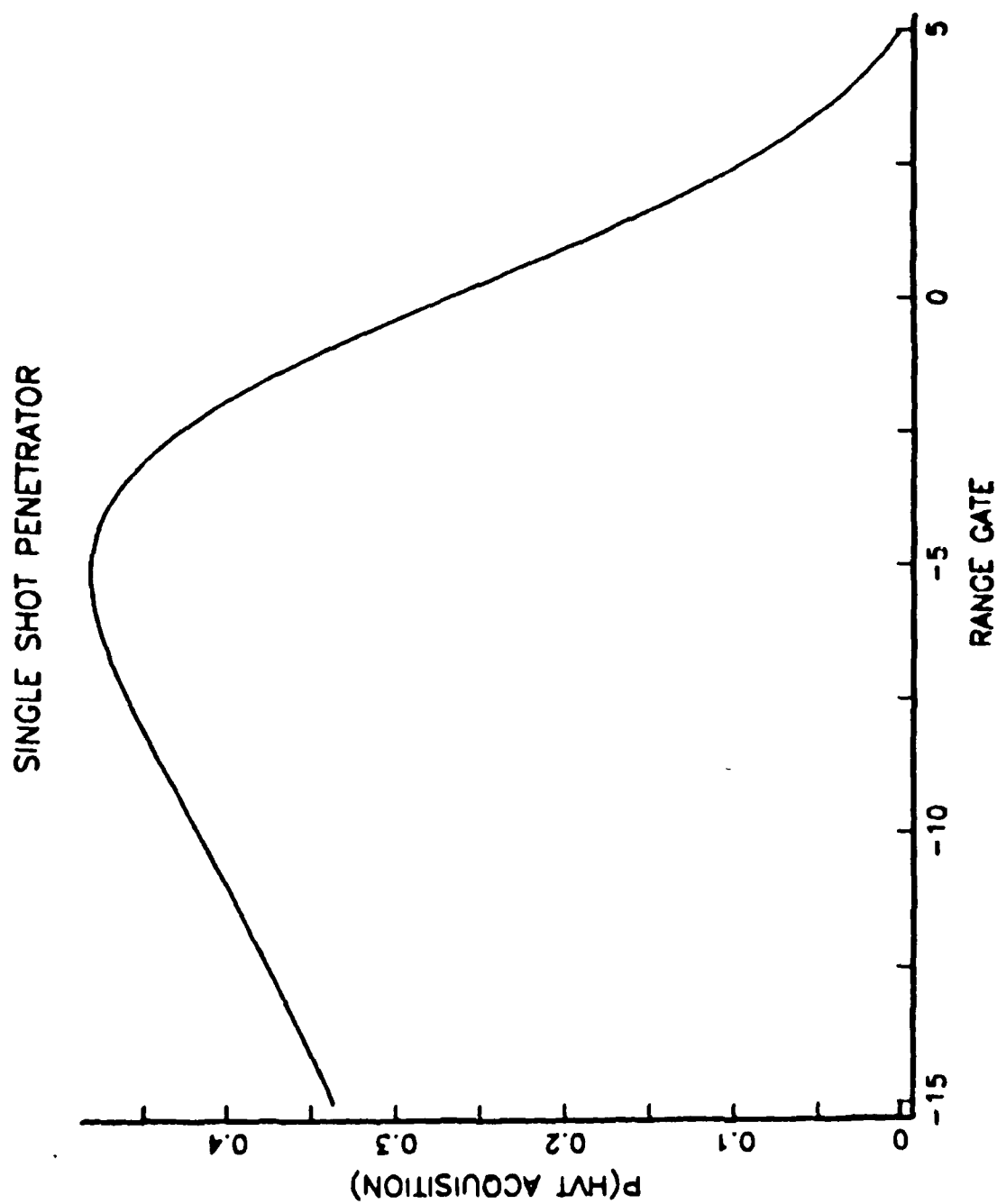


Figure 3.1 SINGLE SHOT P(HVT ACQ.) VS. RANGE GATE.

SINGLE SHOT PENETRATOR

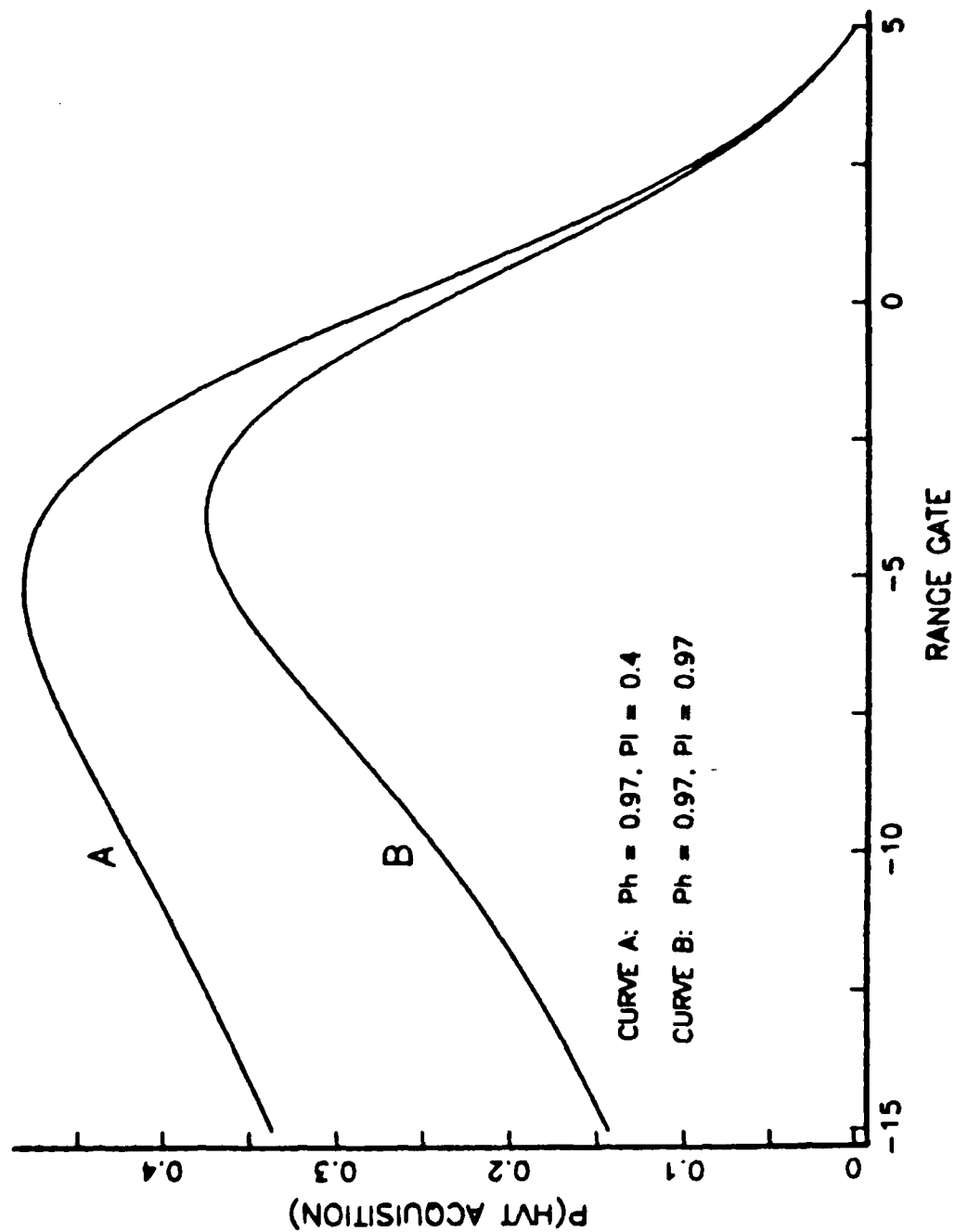


Figure 3.2 THE EFFECT OF MULTI-TARGET DISCRIMINATION.

SINGLE SHOT PENETRATOR VS. FOUR PENETRATORS

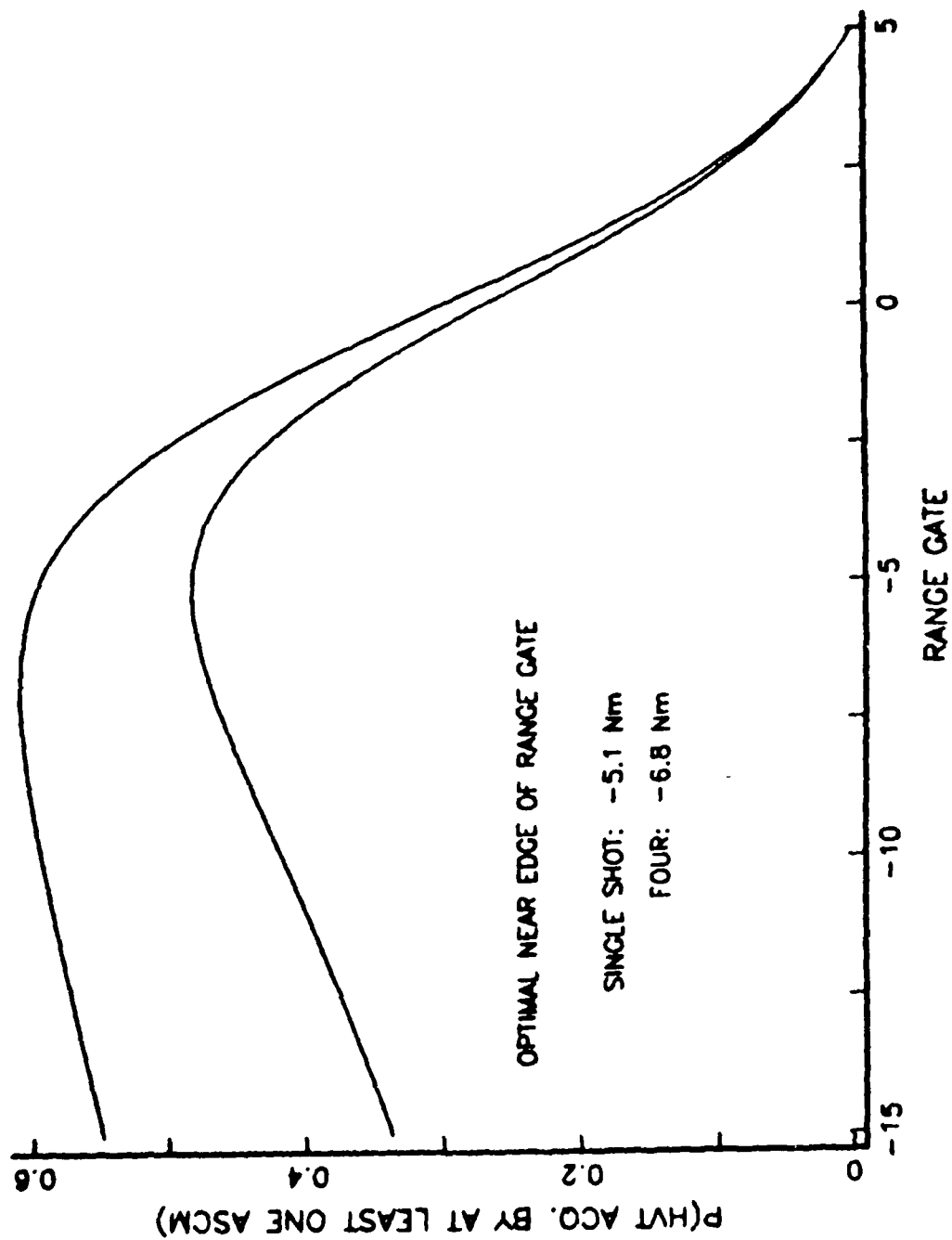


Figure 3.3 ONE VS. MANY.

SINGLE SHOT PENETRATOR VS. TEN PENETRATORS

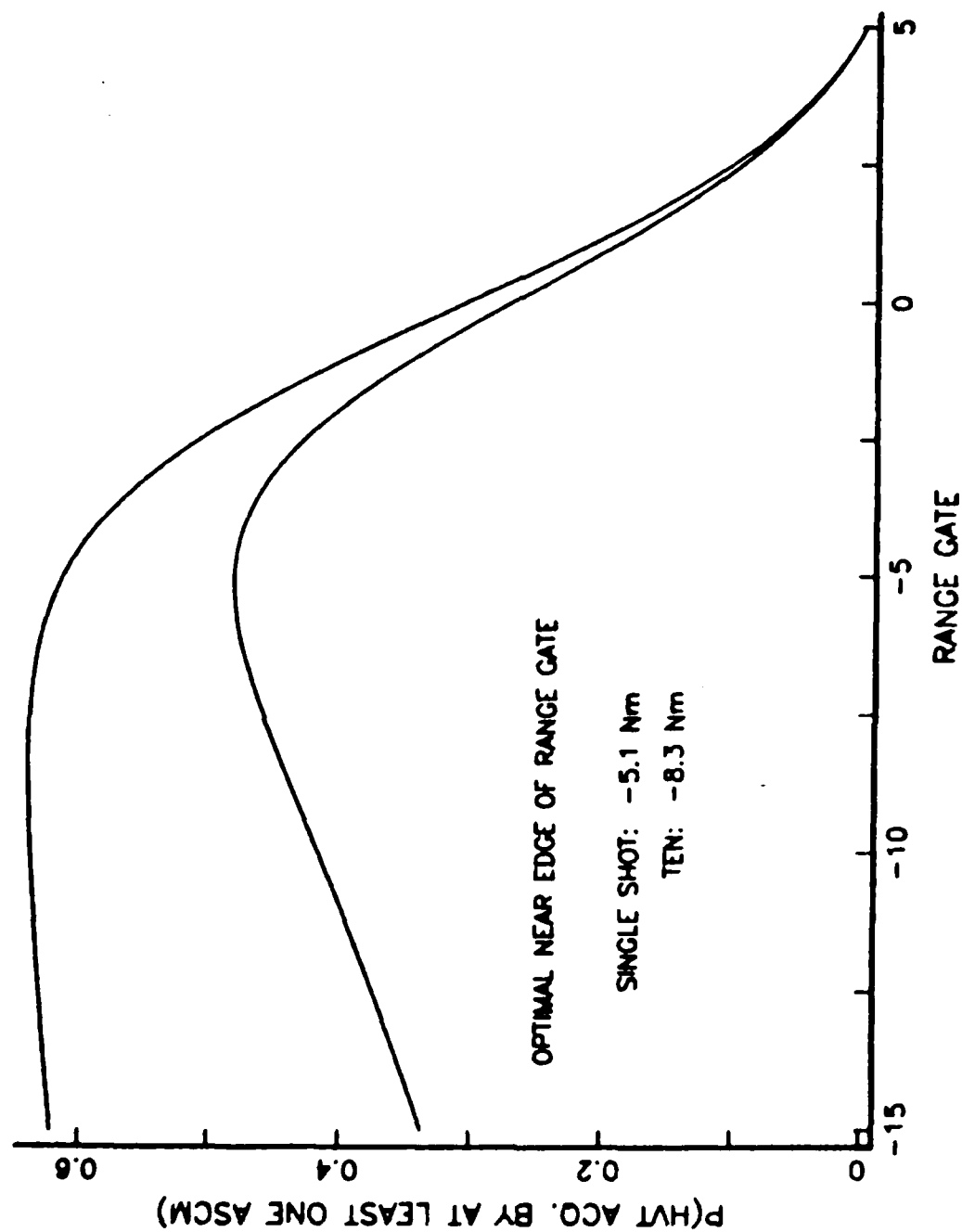


Figure 3.4 RELAXING THE RANGE GATE DECISION VARIABLE.

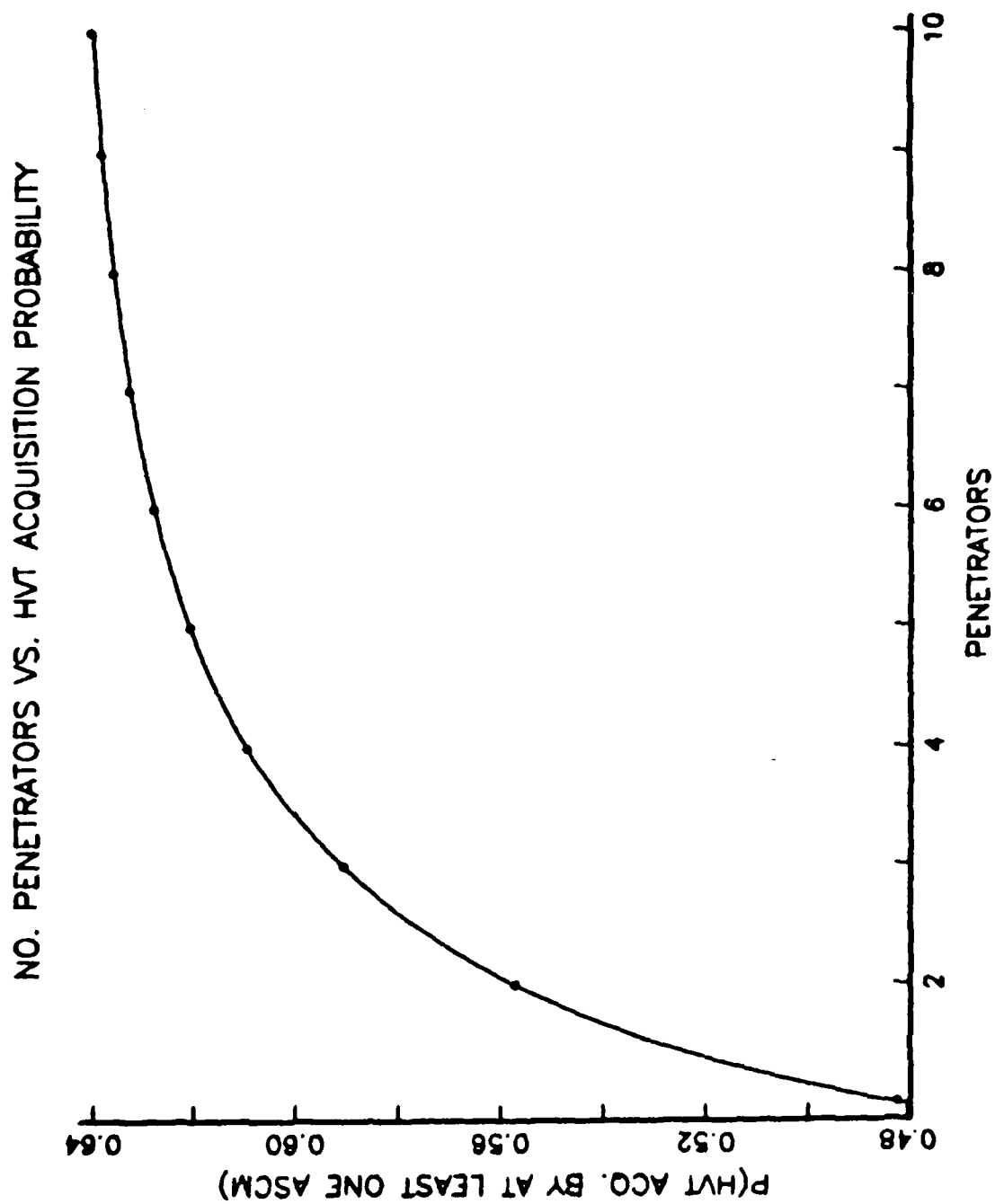


Figure 3.5 INCREASING PENETRATORS.

SINGLE SHOT PENETRATOR

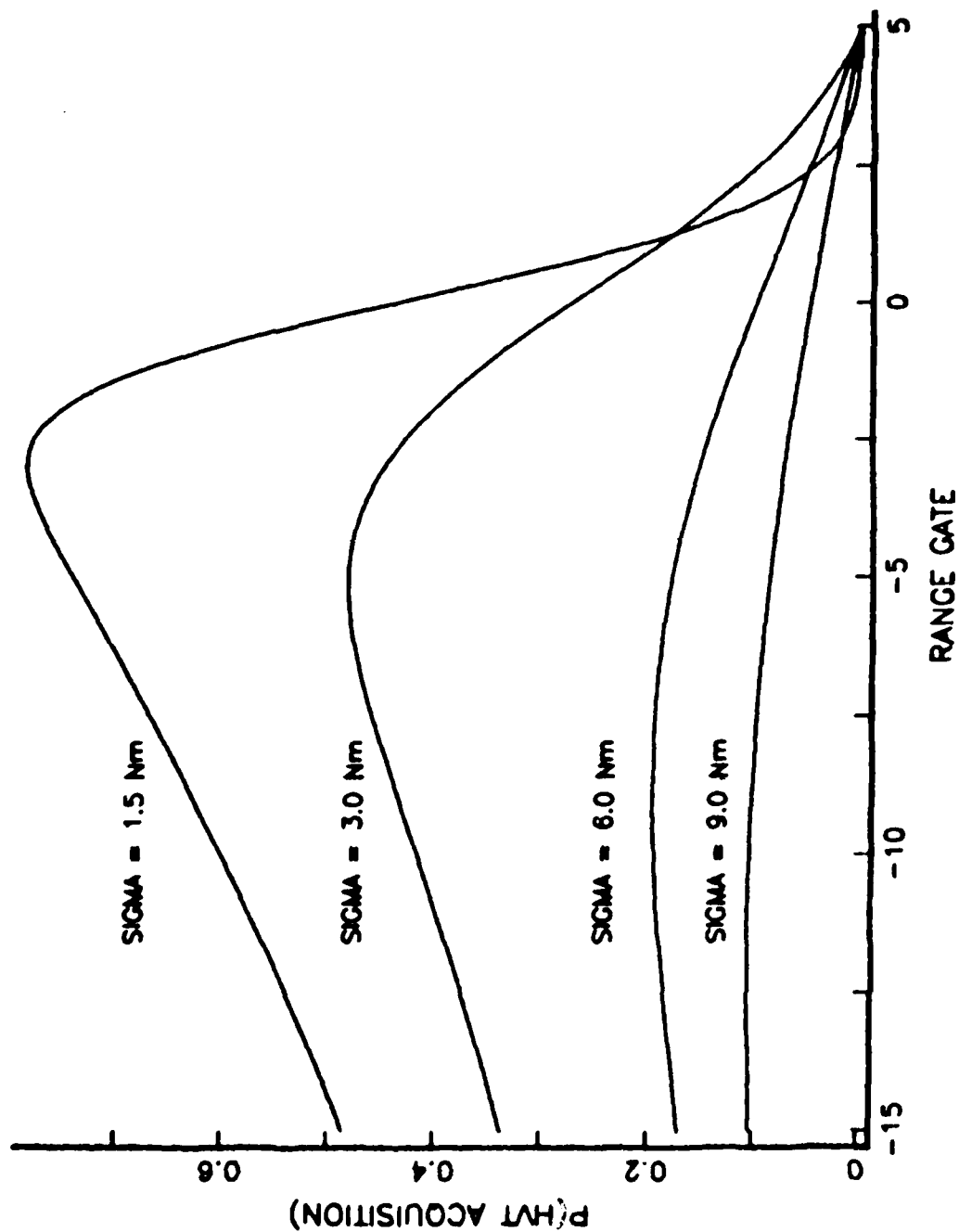


Figure 3.6 TARGETING ACCURACY AND THE SINGLE SHOT.

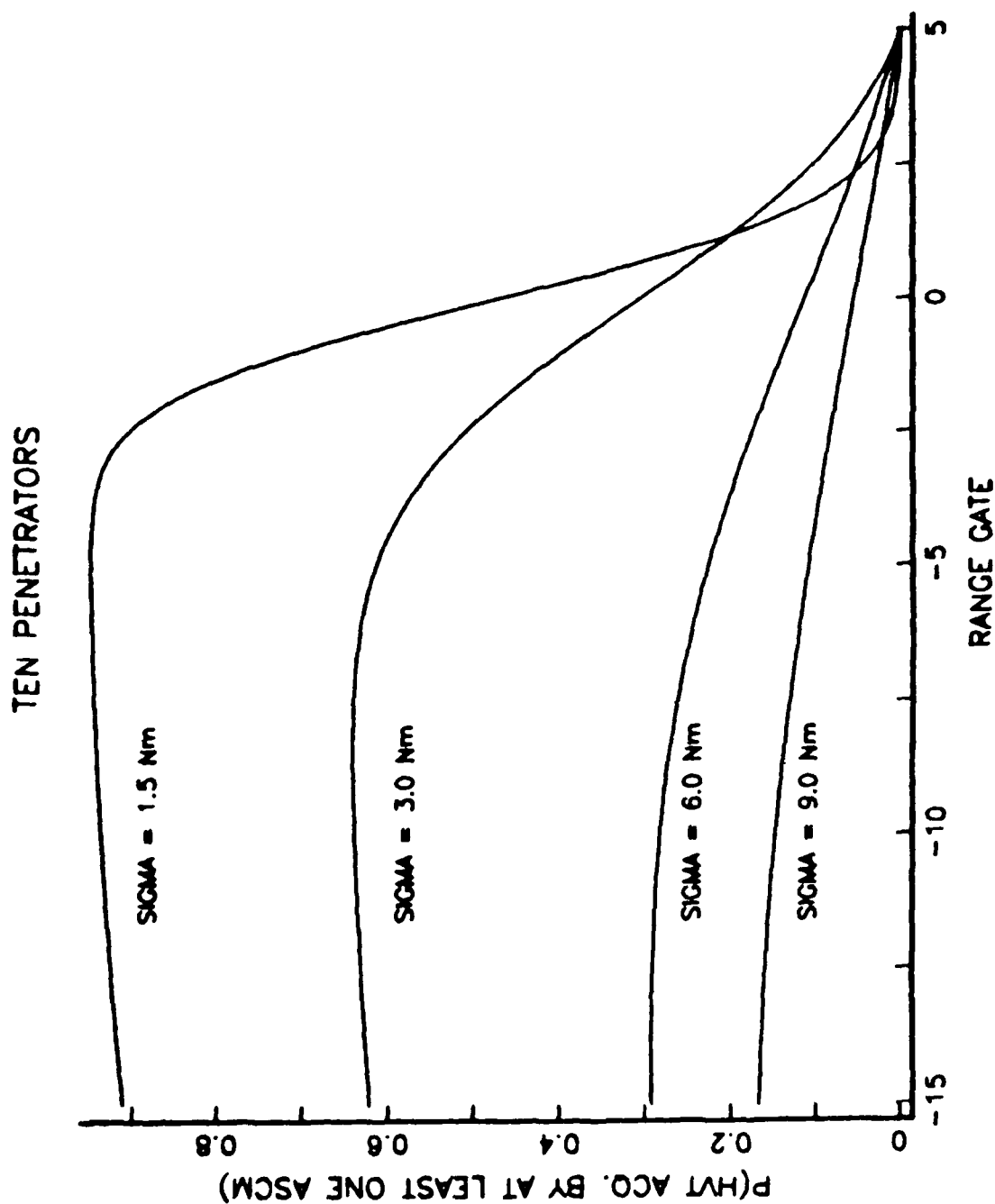


Figure 3.7 TARGETING ACCURACY AND MULTIPLE PENETRATORS.

TABLE I
CONDITIONAL FALL OF SHOT

SINGLE SHOT PENETRATOR

σ	(d_1, d_2)	$P(\text{HVT})^*$	Expectation (Std. dev.) of events:			
			A	B	C	D
1.5	-3.0, +5.0	0.932	0.119	0.844 (0.362)	0.004	0.031
3.0	-5.1, +5.0	0.619	0.188	0.777 (0.415)	0.004	0.029
9.0	-12.1, +5.0	0.162	0.315	0.654 (0.475)	0.005	0.024

FOUR PENETRATORS

σ	(d_1, d_2)	$P(\text{HVT})^*$	Expectation (Std. dev.) of events:			
			A	B	C	D
1.5	-4.0, +5.0	0.950	0.614	3.24 (0.801)	0.019	0.121
3.0	-6.8, +5.0	0.642	0.947	2.92 (1.000)	0.018	0.111
9.0	-16.0, +5.0	0.175	1.57	2.31 (1.310)	0.022	0.088

TEN PENETRATORS

σ	(d_1, d_2)	$P(\text{HVT})^*$	Expectation (Std. dev.) of events:			
			A	B	C	D
1.5	-4.8, +5.0	0.953	1.81	7.84 (1.38)	0.046	0.295
3.0	-8.3, +5.0	0.648	2.81	6.87 (1.89)	0.043	0.268
9.0	-19.1, +5.0	0.181	4.54	5.19 (2.66)	0.053	0.206

* $P(\text{HVT})$ = "The prob. of HVT containment in the search area".

IV. SUMMARY

The number of ASCM area defense penetrators necessary to achieve a required minimum number of HVT acquisitions is a random variable. Tables or nomographs containing number of penetrators, optimal range gate, expected number of HVT acquisitions, and standard deviation of HVT acquisitions versus escort field density could be generated for a set of different posterior distributions describing HVT location over a range of Poisson field densities. It would be necessary to accompany these "conditional" estimates with a probability statement of HVT containment in the effective search area. The model in its present form serves best as an example of how a probability model incorporating a relatively simple analytic form can be used for OTH strike planning. Complex firing situations and target coverage problems involving multiple launches from a consortium of strike groups could be addressed by expanding the simple analytical framework of the model. The equations of a well-tailored (and well-validated) follow-on model could be contained in an interactive computer program which accepts periodic targeting updates and in turn furnishes the strike planner with new estimates of salvo size and optimal range gating to achieve a desired probability of HVT acquisition based on the number of area defense penetrators.

APPENDIX A UPDATING THE HVT LOCATION DENSITY

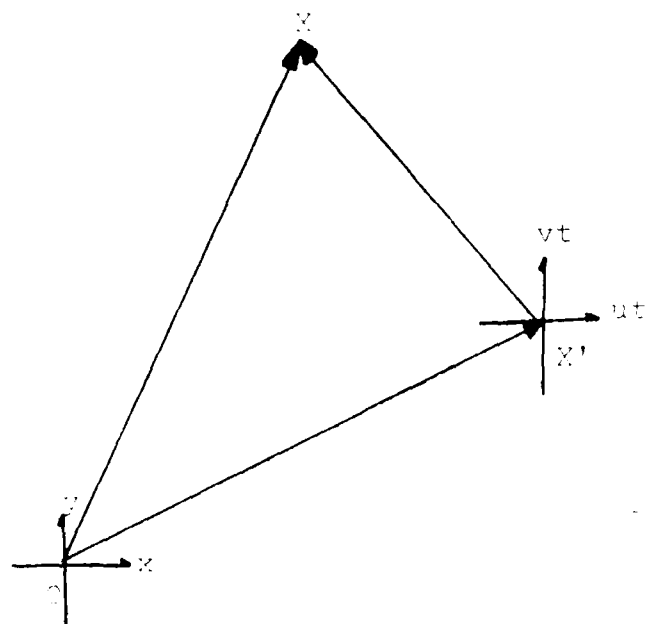
The time delay measured from the generation of the HVT location density to the time the missile commences its search will often allow considerable movement of the enemy formation. A postulated bivariate normal target velocity-time vector can be used to revise the "old" HVT density. The approach does not win "something for nothing" since an assumption concerning enemy intentions is required. Constant target motion is a reasonable assumption if (a) the enemy has not been alerted by the opposition's tracking/targeting methods and (b) timely early-warning of the impending strike is unlikely. A Brownian motion or fleeing datum assumption would be more appropriate in the alerted enemy situation. Dispersion of the enemy formation is also a possibility in the event of a probable nuclear exchange. The constant motion model developed below may prove valid when offensive targeting is reasonably passive and covert.

Let OX , the HVT's vector of movement to its future position measured from the mean of the "old" HVT location density, be the resultant of the sum of two independent bivariate normal vectors: a position error, OX' , and a velocity-time vector $X'X$, (see Figure A.1). The distribution of OX' is given by the HVT density,

$$OX' \sim BVN(\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \rho_{XY})$$

while $X'X$ is distributed as follows,

$$X'X \sim BVN(\mu_U t, \mu_V t, \sigma_U^2 t^2, \sigma_V^2 t^2, \rho_{UV})$$



$$\vec{OX} = \vec{OX'} + \vec{X'X}$$

Figure A.1 THE SUM OF TWO INDEPENDENT RANDOM VECTORS.

The distribution of OX is gleaned from its moment generating function formed by the product of the individual moment generating functions of OX' and $X'X$. The moment generating function of the bivariate normal distribution is:

$$m_{X,Y}(s_1, s_2) = \exp\{s_1\mu_X + s_2\mu_Y + \frac{1}{2}(s_1^2\sigma_X^2 + 2\rho_{XY}s_1s_2\sigma_X\sigma_Y + s_2^2\sigma_Y^2)\}.$$

Therefore,

$$m_{OX'}(s_1, s_2) = \exp\{s_1\mu_X + s_2\mu_Y + \frac{1}{2}(s_1^2\sigma_X^2 + 2\rho_{XY}s_1s_2\sigma_X\sigma_Y + s_2^2\sigma_Y^2)\},$$

$$m_{X'X}(s_1, s_2) = \exp\{s_1\mu_U t + s_2\mu_V t + \frac{1}{2}(s_1^2\sigma_U^2 t^2 + 2\rho_{UV}s_1s_2\sigma_U\sigma_V t + s_2^2\sigma_V^2 t^2)\},$$

and,

$$\begin{aligned} m_{OX}(s_1, s_2) &= m_{OX'}(s_1, s_2) m_{X'X}(s_1, s_2), \\ &= \exp\{s_1(\mu_X + \mu_U t) + s_2(\mu_Y + \mu_V t) \\ &\quad + \frac{1}{2}(s_1^2(\sigma_X^2 + \sigma_U^2 t^2) + 2s_1s_2(\rho_{XY}\sigma_X\sigma_Y + \rho_{UV}\sigma_U\sigma_V t) + s_2^2(\sigma_Y^2 + \sigma_V^2 t^2))\}. \end{aligned}$$

By inspection,

$$OX \sim \text{BVN}(\mu_X + \mu_U t, \mu_Y + \mu_V t, \sigma_X^2 + \sigma_U^2 t^2, \sigma_Y^2 + \sigma_V^2 t^2,$$

$$\rho_{XY}\sigma_X\sigma_Y + \rho_{UV}\sigma_U\sigma_V t).$$

The distribution of OX can be used to adjust the launch bearing of the ASCM salvo and update HVT positional uncertainty to the estimated time of missile search. Model geometry becomes more complicated when a general bivariate normal HVT location density is incorporated instead of a circular normal one since a unique orientation of the effective missile search area results.

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